# Graph theoretical properties of functional and effective connectivity matrix

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The study of the structure of real dynamical (complex) systems is a subject of research in various scientific disciplines such as neuroscience, climatology, sociology, biology or computer science. A key principle in complex network research is viewing the system at hand as a network of interacting subsystems (nodes), with one of the central questions being that of estimating the pattern of mutual or causal interactions of these. While in some cases (computer and social networks), the existence of links can be naturally defined, in other systems (neuroscience, climatology) it is often problematic to determine this structure by direct observation. In this context, the methodology of inference of functional (or effective) connectivity structure using only the knowledge of time series has recently been developed. Such networks are often described by global graph-theoretical characteristics, for example: clustering coefficient, characteristic path length, small-world property, efficiency, transitivity, rich-club coefficient, assortativity and many others.

In practice, Pearson linear correlation is the most commonly used approach to network construction. However, it has been shown that using correlation matrix as connectivity matrix of the system can affect resulting graph-theoretical properties. This phenomenon was described in detail especially for small-world property (coefficient). Using of correlation to graph construction leads, in particular with the knowledge of only a small sample of time series, to the false detection of the small-world property even at random graphs.

Alternative methods to the network construction which should mitigate this problem were suggested containing partial correlation, Granger causality or information theoretical approach. In our work we study the graph-theoretical properties, especially small-world coefficient and rich club coefficient, of effective connectivity matrix in dependence on the form of graph construction. containing (partial) correlation, Granger causality.

Although partial correlation has been proposed as a method for graph construction which should solve the problem with false detection of small-world property, it turns out that partial correlation graphs constructed from relatively short time series has a anti small-world word property (values of small-world coefficient much smaller than random graph). For sufficiently long time series we observe similar behavior as in correlation graphs. We also focused on other graph-theoretical properties especially on recently very discussed rich club coefficient. In both approaches (correlation and partial correlation) we see similar phenomenon - using (partial) correlation itself causes detection of rich-club structure.

A possible solution of overcoming this bias seems to be representing complex systems by its causal structure. The simulations suggest that this family of methods, containing Granger causality or transfer entropy, do not affect the graph theoretical properties but for sufficient estimate of the graph relatively long time series are needed.

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