Functions, universal with respect to classical systems

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In this talk we will construct functions such that their universality is manifested in the classical systems through their Fourier series.

The purpose to describe the structure of functions that are universal for $L^p(0,1)$-spaces, $0 \le p < 1$,with respect to the signs of Fourier-Walsh coefficients. We present some new results.

Theorem 1. There exists such a (universal) function $U \in L^1[0,1)$ and signs $\delta_k = \pm 1$ such that the series $\sum_{|k|=0}^{\infty} \delta_k c_k(U) e^{i2\pi kx}$ is universal in class of all *measurable functions* in common sense,

that is, for each function $f \in M$ *one can find* $\{N_m\}_{m=1}^{\infty} \nearrow$

$$\lim_{m\to\infty} \sum_{0\le|k|\le N_m} \delta_k c_k(U) e^{i2\pi kx} = f(x) \quad \textit{almost everywhere on } [0,1],$$

$$c_k(U) = \int_0^1 U(x) e^{-i2\pi kx} dx, k = 0, \pm 1, \pm 2, \dots .$$

Theorem 2. There exists such a (universal) function $U \in L^1[0,1)$ with strictly decreasing Fourier —Walsh coefficients and converging by $L^1[0,1)$ norm Fourier — Walsh series, the following property:

1) one can find a signs(numbers) $\delta_k = \pm 1$ such that the series $\sum_{k=0}^{\infty} \delta_k d_k(U) W_k(x)$ *is universal, in* $L^p[0,1)$ *for all* $p \in (0,1)$ in common sense,

2) one can find (signs) numbers $\varepsilon_k = \pm 1$ such that the series $\sum_{k=0}^{\infty} \varepsilon_k d_k(U) W_k(x)$ *is universal, in* $L^p[0,1)$ *for all* $p \in [0,1)$ in sense of rearrangements, *that is, for each function* $f \in L^p[0,1)$, $p \in [0,1)$ one can find a permutation $\{\sigma(k)\}_{n=1}^{\infty}$of nonnegative integers, such that the series $\sum_{k=0}^{\infty} \varepsilon_{\sigma(k)} d_{\sigma(k)}(U) W_{\sigma(k)}(x)$ converges to f in *the* $L^p[0,1)$ metric (where $d_k(U) = \int_0^1 U(x) W_k(x) dx$).

Remark. According to the Kolmogorov's theorem (the Fourier series of any integrable function converges in $L^p[0,1)$, $p \in (0,1)$) there is no an integrable function which Fourier series with respect to the trigonometric system is universal in common sense.

References

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