# Extended Constant Returns to Scale in Macroeconomics

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In economics, firms are regarded as economic agents who produce products by inputting production factors such as capital and labor. The production activity of firms is modeled as a function that inputs production factors and outputs total output, and in economics this is called production function. As a simplified model, economists often consider the two-variable function whose production factors are capital and labor. A representative example is the Cobb-Douglas production function [1]. Cobb-Douglas production function is described as the product of the power of capital and the power of labor. Meanwhile, it is well known that capital, labor, and products of firms follow the power law in the large-scale ranges [2]-[4]. This similarity has been discussed earlier [5]. In this research, by interpreting the Cobb-Douglas production function as the quasi-inverse symmetric plane and the residual in the input and the output amount data space [6]-[10], we derived constant returns on input and output normalized by the power indices. Normalized constant returns to scale integrates constant returns to scale, increasing returns, and diminishing returns, which have been discussed in economics. Furthermore, we confirmed the establishment of normalized constant returns numerically from 2005 to 2014 in five countries, Japan, Italy, Germany, the United Kingdom, and Spain.

## References

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