Structural Invertibility and Optimal Sensor Node Placement for Error and Input Reconstruction in Dynamic Systems

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ODE systems are routinely used for the purpose of predicting or controlling complex dynamic systems. However, our knowledge about most real world systems is limited and the system might be perturbed by external influences beyond our control. Reconstructing such unknown inputs from measurements is an important goal in order to observe the state of the system and to predict its future behaviour or to diagnose errors or attacks.

If the inputs can be reconstructed from measurements, we call such a system invertible. We analyse, how invertibility is related to the intrinsic network structure of the system. We find, that homogeneous networks undergo a transition from non-invertible to invertible (see Fig. 1(a)). We also found, that many real systems have a tendency to mask the inputs received. Therefore, invertibility requires a careful selection of outputs which need to be monitored by measurement devices. Importantly, we present a simple yet efficient sensor node placement algorithm to achieve invertibility of complex dynamic systems with a minimum of measurements (see Fig. 1(b)). These results are useful for the development of more realistic mathematical models, for the design of synthetic systems, and for the diagnosis of error or attacks with a minimum set of sensors.

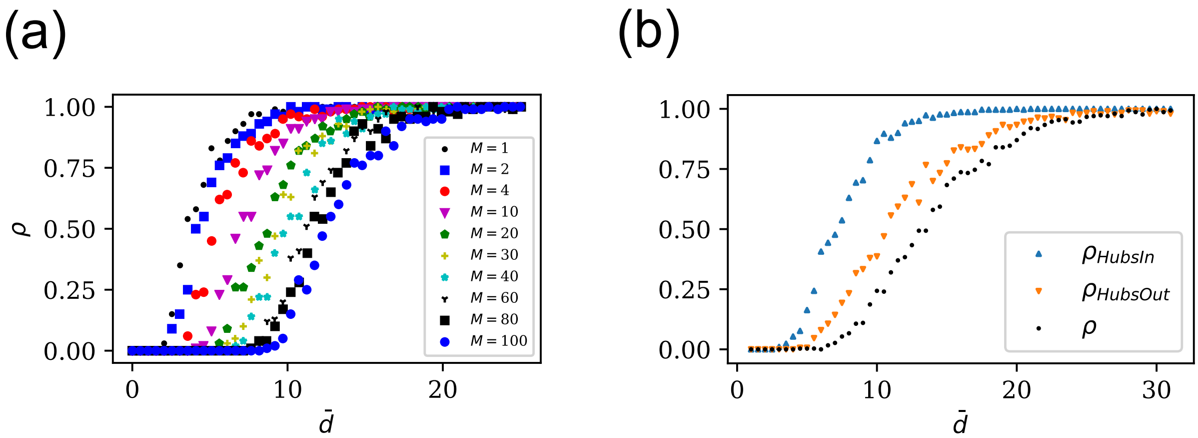

Figure 1: (a) The probability ρ, that a network with M in- and outputs is invertible, shows a phase transition from non-invertible to invertible as a function of the average degree of the network. (b) In a scale-free network, the existence of hubs diminish the invertibility (ρ). By preferential selection of hubs as output nodes (ρ_{HubsOut}) and as input nodes (ρ_{HubsIn}) the invertibility can be increased significantly.

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