Fractal Dimension as a Useful Tool for EEG Analysis

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In 1981 Takens showed that, if we can measure any single variable of a dynamical system with sufficient accuracy, then it is possible to reconstruct a state portrait, topologically equivalent to the attractor of the original system [1]. Complexity of the reconstructed attractor, assessed by the correlation dimension (D_2) is related to the number of variables needed to model the system and - if noninteger - it indicates fractal character of the strange chaotic attractor. In the 1980s, many believed that the low noninteger values of D_2 estimated from EEG were indications of chaos in brain. However, in 1989 Osborne and Provenzale found that also for stochastic signals with a $1/f^\beta$ power spectrum the numerical estimate of D_2 is a small finite value $D_2 = 2/(\beta - 1)$ for $\beta \in (1, 3)$ [2]. These noises as processes with power-law distributions are fractal signals. They are self-affine and hence scale-free. To characterize them, we can use other type of dimension, represented here by Higuchi dimension (HD), which rates irregularity of a curve by a value between 1 and 2 [3]. The measures are elegantly connected: $HD = \frac{5-\beta}{2} = 2 - \frac{1}{D_2}$ [4].
Up to now, a convincing demonstration of chaos has only been obtained at the level of neurons, acting as coupled oscillators. In the case of observing low values of D2 for EEG, the hypothesis of scale-invariant fractal-like structures is preferred rather than the suggestion of deterministic chaos. However, the generative mechanisms of scale-free brain activity are still unclear [5]. What is obvious for now is that fractal dimensions capture fundamental properties of the underlying system. This results in the success in classification of sleep stages, and in distinguishing between normal and pathological or otherwise differing brain states. We also use fractal dimension in detection of directed causal interactions between cortical regions. The method utilizes the fact that the driven dynamics is more complex than the driving subsystem alone.

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